MOTION OF TRIPLE CONFIGURATION OF SHOCK WAVES WITH FORMATION OF WAKE BEHIND BRANCHING POINT

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If three developing shock-wave fronts come together at one point the laws of conservation connecting the parameters of the gas in the vicinity of this point give an overdetermined system of equations. To remove the possible contradiction it is necessary to increase the number of initial parameters. As a rule, the assumption of the presence of a contact discontinuity emerging from the branching point is sufficient. It is also possible for two contact discontinuities to develop which form two shock waves with respect to the branching point opposite the boundary of the isobaric region filled with gas in a state of rest. Such a region is called the wake of the triple point by analogy with the aerodynamic wake for flow around bodies with flow separation. A closed system of five simultaneous differential equations which describes approximately the dynamics of a stream containing a branched system of shock waves with a developing wake behind the triple point is derived and discussed in the report.

1. A wake does not develop behind a triple point under stationary conditions because the small shearing stresses at its boundary cannot be balanced, and in any actual flow such a formation is torn from the wave configuration, rolled into vortices, and carried off by the flow.

In the motion of a triple configuration with an acceleration directed along the relative velocity vector of the impinging stream in front of the branching point the resultant of the shearing stresses at the boundary of the wake can be compensated for by forces of inertia and a wake filled with vortices can develop owing to the resupply with mass from the external stream. One part of the external stream undergoes a single compression at the shock-wave front, the other passes through two shock waves. These effects on the stream lead to the same increase in pressure. Therefore in the stream with a single compression the wave must be stronger close to the direct shock wave.

If all the waves have finite intensity the dynamic pressure in the stream undergoing two-stage compression is many times greater than the dynamic pressure behind a powerful shock wave. This means that the role of the first of the indicated streams with respect to the second approaches the effect of a solid wall. The boundary with the high-pressure stream becomes almost straight and the liberation of space for the developing wake takes place mainly through deformation of the stream passing through the almost straight shock wave. The boundary of this stream becomes substantially curved and the gas penetrating into the region of the wake moves mainly along it. These properties of the flow structure are reflected in Fig. 1, where the shock-wave fronts are drawn with solid lines, the contact discontinuities with dashed lines, and the characteristics with dash–dot lines, $\mathbf{v}$ is the velocity of the impinging stream, and D is the drift rate of the branching point.

In the case under examination of the relative orientation of the velocity of the impinging stream and the acceleration of the branching point the intensity of the wave close to the direct shock wave decreases with time. In the absence of a wake (a particular case is where the two boundaries of the wake merge into a single contact discontinuity) a relatively narrow zone of rapid variation in the parameters forms behind the front of such a wave. In the presence of an almost isobaric wake such a zone separates from the wave

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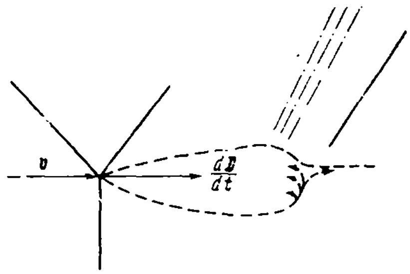

Fig. 1

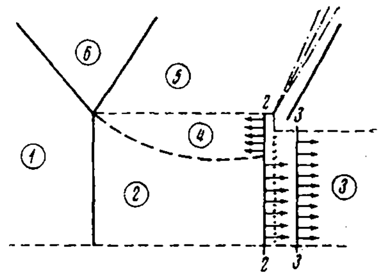

Fig. 2

and moves behind it at a constant distance equal to the length of the wake. The presence of a section of sharp variation in the flow geometry in the "tail" of the wake makes it possible to simulate this effect by a break in the parameters with the inclusion among them of the area of the cross section of the stream and the mass flow rate, since the principal drawing off of mass into the region of the wake is concentrated precisely in the tail section.

The enumerated structural properties of the flow under consideration are reflected in the following idealized flow diagram. In Fig. 2 region 1 is continuous, generally speaking, nonuniform flow – the impinging stream; 2 is unsteady flow with subsonic velocity relative to the shock wave, which can be considered in a one-dimensional approximation as having a cross-sectional area which varies along the coordinate and with time. The flow in this region has the nature of entropy waves generated by a powerful transient shock wave; region 3 is analogous to 2 but with simpler properties: here the cross-sectional area depends only on time and not on the longitudinal coordinate. Contact between regions 2 and 3 is made through a conditional discontinuity satisfying the conservation laws. The arrows show the directions of the velocities with respect to the moving discontinuity. The supply of mass to the region of the wake is accomplished in the section where the arrows turn around; 4 is the region of flow in the wake; 5 and 6 are regions of quasi-stationary supersonic uniform flows of gas.

A closed system of equations for the qualitative flow diagram characterized is derived and discussed below.

2. For regions of continuous gas flow in a streamtube passing through a strong shock wave an analytical description of the flow is possible in a one-dimensional approximation with a cross-sectional area F of the stream which depends on the time t and the longitudinal coordinate x. The system of equations of motion is written in the form

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}+\frac{1}{\rho}\frac{\partial p}{\partial x}=0, \quad \frac{\partial \rho F}{\partial t}+\frac{\partial \rho v F}{\partial x}=0$$

$$\frac{\partial S}{\partial t}+v\frac{\partial S}{\partial x}=0, \quad S=\frac{p^{1/k}}{\rho}$$

Here v is the velocity, p is the pressure, ρ is the density, S is the entropy function, and k is the ratio of heat capacities. According to the simplified system of flow adopted (Fig. 2), in the sections of continuous flow the pressure is a function only of the time ($\partial p/\partial x = 0$). With this condition the system of equations written can be integrated. The general solution is obtained in the form

$$p^{1/k}F[1+tV'(\lambda)]=\beta(\lambda), \quad v=V(\lambda), \quad S=S(\lambda), \quad \lambda=x-vt, \quad p=p(t) \tag{2.1}$$

where β, V, and S are arbitrary functions of the argument λ.

If the area F of the stream cross section depends only on time, as occurs for region 3, then two arbitrary functions, namely β and V, are determined with the accuracy of arbitrary constants. In this case it follows from (2.1) that

$$V'(\lambda)\equiv \mathrm{const}=A, \quad \beta(\lambda)=\mathrm{const}=C$$

Thus, for region 3

$$F = F(t), \quad p^{1/k} F(1 + At) = C, \quad v = \frac{Ax + B}{At + 1}, \quad \lambda = \frac{x - Bt}{1 + At} \tag{2.2}$$

where A, B, and C are arbitrary constants.

3. In accordance with the model flow diagram illustrated in Fig. 2, a break in the parameters of the stream is conditionally established in the cross section where the wake breaks off. Since such a break simulates a rapid but continuous change in parameters, the process of transition through the dicontinuity can be considered as isentropic. We can take two adjacent cross sections (2−2 and 3−3) before and after the simulated discontinuity, respectively. Considering an internal flow with a high dynamic pressure, one can assume a change in the cross-sectional area of the stream during passage through this discontinuity only in the direction of a decrease. In this case the external supersonic stream, flowing around the conditional step (Fig. 2), passes through two waves, of rarefaction and compression, of practically equal intensity, although the turning angles of the stream in these waves must be different.

Full recovery of the pressure cannot occur if the compression wave represents a sufficiently strong shock wave, which apparently must be ruled out under the conditions being examined. Then one can assume the equality of the pressures on both sides of the discontinuity. The equality of the densities in the cross sections 2−2 and 3−3 follows from this and from the assumption of isentropy. Thus, sharp changes between these cross sections occur in the velocity, cross-sectional area of the stream, and mass flow rate, which changes because of the supplying of mass to the wake. We assume that the gas particles enter the wake without additional irreversible losses. Then it follows from the equality of entropy and pressure for these particles that the densities in them are equal to the densities ρ_2 in the cross section 2−2 of the main stream.

Let F be the area of the shock wave at the base of region 2, N be the drift rate of the conditional shock wave, and q be the influx of mass into the wake. We will mark the parameters in cross sections 2−2 and 3−3 with the indices 2 and 3. The equations for the mass flow rates are

$$Q_2 = \rho_2 F_2 (v_2 - N), \quad q = \rho_2 (F - F_2)(N - D), \qquad Q_3 = \rho_2 F_3 (v_3 - N) \tag{3.1}$$

The corresponding total energy densities are

$$\varepsilon_2 = \frac{v_2^2}{2} + \frac{1}{k-1}\frac{p_2}{\rho_2}, \quad \varepsilon = \frac{D^2}{2} + \frac{1}{k-1}\frac{p_2}{\rho_2}, \quad \varepsilon_3 = \frac{v_3^2}{2} + \frac{1}{k-1}\frac{p_2}{\rho_2}$$

Let p′ be the pressure at the contact discontinuity between cross sections 2−2 and 3−3. Let us write the conditions of compatibility which follow from the conservation laws:

$$Q_2 = Q_3 + q, \quad Q_2 v_2 + p_2 F = Q_3 F_3 + p_3 F_3 + qD + p'(F - F_3)$$

$$\varepsilon_2 Q_2 + p_2 v_2 F_2 = Q_3 \varepsilon_3 + p_3 v_3 F_3 + q\varepsilon - p_2 D (F - F_2) + p'(F - F_3) N$$

Transformations with allowance for the assumptions made result in the following convenient form of the latter equations:

$$\begin{gathered} Q_2 - Q_3 - q = 0 \\ Q_2 (v_2 - N) - Q_3 (v_3 - N) - q(D - N) = (p_2 - p')(F_3 - F) \\ Q_2 (v_2 - N)^2 - Q_3 (v_3 - N)^2 - q(D - N)^2 = 0 \end{gathered} \tag{3.2}$$

4. In a stationary impinging stream the pressure is a known function of the coordinates p = p(x, y), in the flow behind the front p = p(t), and the entropy in the impinging stream is S_1 = const. From the relationships at the shock wave the drift rate D of the wave can be expressed through the velocity V of the gas behind the front and the Mach number M_1 in the impinging stream. Considering the strongest wave in the triple configuration as a direct shock wave one can write three independent equations for the gas passing through this wave front. To simplify the notation we will use approximate equations, assuming that $1/M^2 \ll 1$ and $|D/v_m| \ll 1$, where v_m is the maximum velocity of steady discharge into a vacuum:

$$\begin{gathered} D = \frac{k+1}{2} V(\lambda) - \frac{k-1}{2} v_m, \quad \frac{p(t)}{p(xy)} = \frac{2k}{k+1} M_1^2 \left(1 - 2\frac{D}{v_m}\right), \\ \frac{S(\lambda)}{S_1} = \frac{k-1}{k+1}\left(\frac{2k}{k-1}\right)^{1/k} M_1^{2/k}\left(1 - \frac{2}{k}\frac{D}{v_m}\right) \end{gathered} \tag{4.1}$$

5. Let us examine the conditions which account for the fact of the existence of a triple configuration of shock waves. We shall direct the X axis along the velocity vector in region 5 (Fig. 2). Let ϑ be the angle of inclination of the velocity vector to the X axis in front of the triple point, ϑ_1 be the turning angle of the stream at the wave front between regions 1 and 6, and ϑ_6 be the analogous angle for the shock wave separating regions 5 and 6. Then

$$\vartheta + \vartheta_1 + \vartheta_6 = 0 \tag{5.1}$$

For the flow diagram illustrated in Fig. 2 the construction of a triple configuration with a given ϑ which satisfies the condition (5.1) comes down to the determination of the roots of a transcendental equation. This is possible in each specific case but it is difficult to use such a procedure as a boundary condition. Simplification of the calculating equations is achieved by linearization with respect to a triple configuration with one direct shock wave [1]. Let us examine this procedure. Let us introduce into the function under consideration two variables which we designate as P and R and their arguments which we designate as M and Z:

$$\begin{gathered} P(M, z) = \frac{z-1}{1+kM^2-z}\left[\frac{2k(k+1)^{-1}M^2}{z+(k-1)/(k+1)}\right]^{1/2} \\ R(M, z) = \left[M^2\left(\frac{k+1}{k-1}z+1\right)-\frac{2}{k-1}(z^2-1)\right]\left[z\left(z-\frac{k+1}{k-1}\right)\right]^{-1/2} \end{gathered} \tag{5.2}$$

The first argument later has the meaning of Mach numbers while the second argument takes on the meaning of different pressure ratios. Marking the parameters with the numbers of the corresponding regions, we can write

$$\mathrm{tg}\,\vartheta_1 = P(M_1,\ p_6/p_1),\quad \mathrm{tg}\,\vartheta_6 = P(M_6,\ p_5/p_6),\quad M_6^2 = R(M_1,\ p_6/p_1)$$

We will mark with a second index 0 the parameters of the triple configuration in which the powerful shock wave is not approximately but exactly a direct shock wave. Then $\vartheta_{10} + \vartheta_{60} = 0$, or because of the latter equalities

$$\begin{gathered} M_{60}^2 = R(M_1,\ p_{60}/p_1),\quad p_{50}/p_1 = 2kM_1^2/(k+1) - (k-1)/(k+1) \\ \mathrm{arc\,tg}\,P(M_1,\ p_{60}/p_1) - \mathrm{arc\,tg}\,P(M_{60},\ p_{50}/p_{60}) = 0 \end{gathered} \tag{5.3}$$

Equations (5.3) give a transcendental equation for the pressure p_{60} between the fronts of branched shock waves. Henceforth we will assume that this equation is solved, i.e., the dependence $p_{60}/p_1 = f(k, M)$ is determined. For approximate calculations it is convenient to represent this dependence in the form of a polynomial in the two arguments $1/M$ and $(k-1)$. In the ranges of the parameters $1.15 \le k \le 1.67$, $2 \le M \le 10$ an approximation with an accuracy of 3% is given by the quadratic polynomial

$$\begin{gathered} p_{50}/p_{60} = a + b/M + c/M^2,\quad a = -0.07 + 0.77(k-1) + 1.02(k-1)^2 \\ b = 1.74 - 4.86(k-1) + 8.24(k-1)^2,\quad c = -1.19 + 5.85(k-1) - 12.16(k-1)^2 \end{gathered} \tag{5.4}$$

Further, we assume that the angle ϑ is small and the parameters of flow between the shock wave fronts differ little from the corresponding parameters in a configuration with one direct shock wave:

$$p_5 = p_{50} + \Delta p_5,\quad p_6 = p_{60} + \Delta p_6,\quad M_6 = M_{60} + \Delta M_6$$

For the derivatives of the functions (5.2) we introduce the following designations:

$$\begin{gathered} P_1(M, z) = \frac{\partial P}{\partial (M^2)} = \frac{k^2}{k+1}P(M, z)(z+1-M^2)\left[(1+kM^2-z)\left(\frac{2k}{k+1}M^2-z-\frac{k-1}{k+1}\right)\right]^{-1} \\ P_2(M, z) = \frac{\partial P}{\partial z} = P\left(\frac{1}{z-1}+\frac{1}{1+kM^2-z}-\frac{0.5}{z+(k-1)/(k+1)}-\frac{0.5}{2kM^2/(k+1)-z-(k-1)/(k+1)}\right) \\ R_2(M, z) = \frac{\partial R}{\partial z} = -\left(M^2+\frac{2}{k-1}\right)\left[\frac{k+1}{k-1}(z^2+1)+2z\right]\left[z\left(\frac{k+1}{k-1}+z\right)\right]^{-2} \end{gathered} \tag{5.5}$$

From Eq. (5.1) we find, with an accuracy of the small values of first order, that

$$\Delta p_6/p_1 = \sigma\vartheta$$
$$\sigma = \{\cos^2\vartheta_{60}\,[P_2(M_1, p_{60}/p_1) + P_1(M_{60}, p_{50}/p_{60})\,R_2(M_1, p_{60}/p_1) + P_2(M_{60}, p_{50}/p_{60})\,(p_1/p_{60})^2\,(p_{50}/p_1)]\}^{-1} \tag{5.6}$$

Let us consider the angular coefficient of inclination of the shock wave front between regions 1 and 6: $dy/dx = \mathrm{tg}(\omega - \vartheta)$, where ω is the angle of inclination of the wave front to the local direction of the velocity vector ahead of the front. We assume that $\omega = \omega_0 + \Delta\omega$. Using the connection between the pressure and the angular inclination of the wave

$$p_6/p_1 = \frac{2k}{k+1}M^2\sin^2\omega - \frac{k-1}{k+1}$$

we find with the accuracy adopted above that

$$dy/dx = \mathrm{tg}\,\omega_0 + \frac{k+1}{4kM^2\sin\omega_0\cos^3\omega_0}\frac{\Delta p_6}{p_1} - \frac{\vartheta}{\cos^2\omega_0} \tag{5.7}$$

In the equations used the Mach number is composed from the relative velocity of the stream ahead of the front

$$M = (v_1 - D)/a_1 = M_1 - D/a_1$$

Later in the expansions we retain only the terms linear with respect to D. Then from (5.6) and (5.7) we obtain

$$dy/dx = -K_0 + K_1 D/v_m + K_2\vartheta \tag{5.8}$$
$$K_1 = \frac{k}{k+1}M_1\sqrt{M_1^2 + 2/(k-1)}\left\{M_1\frac{\partial f(k, M_1)}{\partial M_1} - 2\left[f(k, M_1) + \frac{k-1}{k+1}\right]\right\}\left[\frac{2k}{k+1}M_1^2 - f(k, M_1) - (k-1)/(k+1)\right]^{-2}$$
$$K_0 = \mathrm{tg}\,\omega_0|_{D=0}, \quad K_2 = \frac{1}{\cos^2\omega_0}\left[\frac{(k+1)\,p_{60}/p_1}{4kM_1^2\sin\omega_0\cos\omega_0} - 1\right]$$

Equation (5.8) characterizes the essentially non-one-dimensional effects of the triple configuration of waves and closes the system of equations presented above.

6. The full system of equations obtained is written in an inconvenient form for analysis. Let us separate the finite relations from the differential relations and transform the differential equations to the standard form of notation. It is convenient to consider all the unknown values as functions of the parameter λ introduced by Eq. (2.1). Let $x = x(\lambda)$ and $t = t(\lambda)$ be the parametric equations of motion of the shock-wave front and $\xi = \xi(\lambda)$ and $\tau = \tau(\lambda)$ be the coordinates of the point on the trajectory of the simulated discontinuity in the xt plane with the same values of λ as at the shock wave. We will relate the conditions of compatibility (3.1) and (3.2) to this point. Then none of the functions which depend only on λ differs behind the wave front and in the cross section 2–2, i.e., one must set

$$v_2 = V(\lambda),\ S_2 = S(\lambda),\ \beta_2 = \beta(\lambda);\ p(\lambda) \neq p_2(\lambda),\ F(\lambda) \neq F_2(\lambda)$$

since p depends on time but not on λ, and F is a function of both λ and t. The fact that p depends only on the single argument t can be expressed by the following equation:

$$p_2(\lambda) = p[\lambda(t)]\,|_{t=\tau(\lambda)}$$

where $\lambda(t)$ is the values of λ at the shock wave front. Differentiating the latter equation with respect to λ we obtain

$$\frac{dp_2}{d\lambda}\frac{dt}{d\lambda} = \frac{dp}{d\lambda}\frac{d\tau}{d\lambda} \tag{6.1}$$

Because of the first of Eqs. (2.1)

$$\frac{dV}{d\lambda} = \frac{\nu - 1}{t - \nu\tau}, \qquad \nu = \frac{F_2 p_2^{1/k}}{F p^{1/k}} \tag{6.2}$$

Keeping in mind that

$$V = \frac{x-\lambda}{t} = \frac{\xi-\lambda}{\tau}, \qquad \frac{dx}{d\lambda} = D\frac{dt}{d\lambda}$$

and taking into account the differential equations (5.8) and (6.1) and the second equation of (4.1), after transformations we obtain the system of differential equations

$$\frac{dx}{d\lambda} = \frac{D\nu(1-\tau/t)}{(D-V)(1-\nu\tau/t)}, \quad \frac{dt}{d\lambda} = \frac{\nu(1-\tau/t)}{(D-V)(1-\nu\tau/t)}$$

$$\frac{dt}{d\lambda} = \nu\frac{N-V}{D-V}\frac{d\tau}{d\lambda}, \quad \frac{dy}{d\lambda} = \left[-K_0 + K_1\frac{D}{v_m} + K_2\vartheta(x,y)\right]\frac{dx}{d\lambda} \tag{6.3}$$

$$\frac{dp_2}{d\lambda} = \frac{2k^3}{k+1}\frac{D-V}{\nu}\left\{\left(1-\frac{k+1}{kv_m}\frac{x-\xi}{t-\tau}\right)\left[\frac{\partial(p_1M_1^2)}{\partial x}\frac{dx}{d\lambda} + \frac{\partial(p_1M_1^2)}{dy}\frac{dy}{d\lambda}\right] - \frac{k+1}{k}\frac{\nu-1}{t-\nu\tau}p_1M_1^2\right\}$$

The unknown functions here are x, t, y, τ, and p_2. The functions $p_1 = p(x, y)$ and $M_1 = M(x, y)$ characterize the stationary state of motion in the impinging stream and are assumed to be given. The values D and V entering into the coefficients of the system (6.3) are expressed through the unknown functions, since according to the first equation of (4.1)

$$D = \frac{k+1}{2}V - \frac{k-1}{2}v_m = \frac{k+1}{2}\frac{x-\xi}{t-\tau} - \frac{k-1}{2}v_m$$

The values of ν and N in the system (6.3) are not determined explicitly through the unknown functions. The remaining algebraic equations must be used to find the dependences of the required structure: the three equations of (3.2), the three equations of (3.1) for the flow rates, and two equations which follow from the general solution for region 3 in the form of (2.2):

$$v_3 = (A\xi + B)/(A\tau + 1), \qquad p_2^{1/k}F_3(1 + A\tau) = C$$

These eight algebraic equations contain eight unknown values (N, Q_2, F_2, q, Q_3, F_3, v_3, p') which must be found in the form of functions of x, t, y, τ, and p_2. The larger number of unknown values is a defect of the notation. The appropriate simplifications of the system which are connected with the reduction of the unknowns are carried out straightforwardly but are rather cumbersome and are omitted here, especially since each of the eight unknown values enumerated above is an interesting characteristic of the process. The simplification can be reduced, for example, to one quadratic equation relative to N, which is obtained if (3.2) is solved as a linear system with respect to the flow rates and the results are equated with Eqs. (3.1).

The following statements are valid: 1) the structural schematization of the flow adopted is not inconsistent from the point of view of its analytical description; 2) the flow has a high degree of determinacy: the arbitrariness in the solution consists of a few constants.

The latter statement naturally concerns only the "skeleton" of the flow; the three-dimensional distortions of the fields of the parameters and the breakdown of the quasistationary nature of the pressure introduce the proper arbitrarity, but the initial qualitative configuration of the stream is assumed to be stable and the statements formulated pertain only to this configuration.

An approximate solution of the system (6.3) can be obtained by partial freezing of the slowly varying coefficients. Since usually $V \ll v_m$ it follows from the first equation of (6.3) that practically $D - V = \text{const}$. The coefficient ν represents the ratio between the mass flow rates through the cross section where the shock wave is located and the cross section 2−2, taken at the same λ. The coefficient $\nu(N - V)/(D - V)$ characterizes the difference between the flow rates through the same, but moving, cross sections. Neither of these values can undergo large or abrupt changes and over considerable intervals they can be replaced by average values. In such an approximation the solution of system (6.3) is written in the form

$$x = C_1\lambda^{(k+1)2} + \nu_1\lambda + x_0, \quad t = \nu_2\lambda, \quad t = \mu\tau$$

$$\nu_1 = \frac{k+1}{k-1} + \nu_2 v_m, \quad \nu_2 = \frac{\nu(1-\mu)}{(D-V)(1-\nu\mu)}, \quad \mu = \nu\frac{N-V}{D-V} \tag{6.4}$$

where C_1 and x_0 are arbitrary constants. The last two equations of system (6.3) can be interpreted for a specific analytical setting of the state of motion in the undisturbed flow.

7. When an underexpanded supersonic jet impinges on a barrier a complicated stream is formed with a branched system of shock waves and containing regions of local subsonic flow, contact surfaces, and sec-

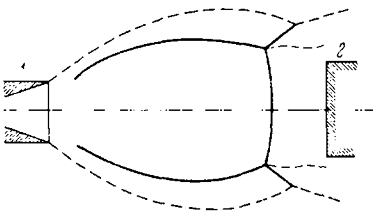

Fig. 3

tions of flow with large gradients of the parameters (see Fig. 3, where 1 is the nozzle, 2 is the barrier, the solid lines are shock waves, and the dashed lines are contact discontinuities). The study of the properties and structure of such streams is complicated by the fact that in certain ranges of the unknown parameters the stationary configuration of the flow becomes unstable, a spontaneous transition from a stationary to a nonstationary stream is realized, and a strongly pulsating process develops in front of the facing side of the barrier [2]. An explanation for the mechanism of these effects is lacking. The pulsations of the stream are accompanied by considerable movements of the powerful central shock wave with respect to the irregular background, as a result of which intense entropy waves pass through the subsonic jet behind this shock wave.

Several discrete vibrational tones differing markedly in frequency are usually observed. The low-frequency pulsations can have a large amplitude. The amplitude of the high-frequency vibrations is usually small and estimates of the frequency show that these vibrations are connected with processes which propagate with the speed of sound.

The first step in constructing a model of the low-frequency cycles consists in ignoring the role of the high-frequency vibrations and their distorting effect on the development of the processes with time. This is achieved if the speed of sound is taken as infinite in the subsonic region behind the central shock wave. In such an approximation the quasistationary nature of the processes in the local subsonic region is disturbed only by the entropy waves. Within the limits of one period the evolution of the wave structure can be described with the help of the schematic wave model examined above. Suppose that at some intermediate position of the central shock wave an arbitrary discontinuity in the gas parameters develops whose decay gives rise to the movement of this shock wave in the direction from the barrier toward the nozzle with some final velocity $D_0 < 0$. The subsequent variation in the velocity of the wave can be determined by using the approximate law (6.4):

$$D = \left(D_0 - \frac{\nu_1}{\nu_2}\right)\left(\frac{t}{t_0}\right)^{(k-1)/2} + \frac{\nu_1}{\nu_2}, \quad \nu_2 < 0, \quad \nu_1 > 0$$

The velocity of the wave decreases with time and at some moment is reduced to zero, after which the return movement of the wave toward the barrier begins. The intensity of the central shock wave decreases monotonically. Therefore the Mach number behind the front increases. At some moment the velocity in front of the barrier becomes supersonic. This leads to the appearance of a shock wave whose intensity increases as it gets farther from the barrier. At the moment this wave encounters the central shock wave an arbitrary discontinuity develops whose decay creates the prerequisites for a repeat cycle. These prerequisites do not arise in two cases: 1) the barrier is located too close to the nozzle cut (supersonic velocities are not reached in the impinging central jet); 2) the barrier is too far away. Supersonic velocity in front of the barrier is reached in a stationary free jet and the barrier does not affect the central shock wave.

A relatively narrow range exists in which the system described allows the development of a self-sustaining vibrational process.

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